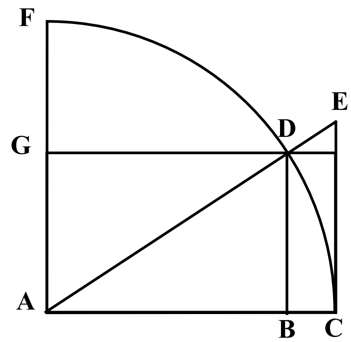
§15.1 Chapter Fifteen

Concerning Tangents and Secants

With the canon of sines for hundredths or thousandth parts of degrees, the canons of tangents, secants, and of logarithms are provided with the same fractional parts.
Prop.1. Tangents and secants are most conveniently found by the Rule of Proportion. For any sine is to the sine of the complement : as The radius to the tangent of the same complement.

CD	The Arc of 35:0': Degrees		
DF	55:0.		
Pro-port.	AB Sine	55:0'	81915
	BD Sine	35:0'	57350
	AC Radius		100000
	CE Tangent	35:0' . . .	70021

[Table 15-1]



[Figure 15-1]

By this Proposition alone any of the whole quadrant of tangents can be found.
Prop. 2. The radius is the mean proportional between any of these as you please the arcs of the sine and of the complement of the secant.

	AB Sine	55:0'	81915
Pro-	AD Radius	. . . . ….	100000
port.	AC Radius		100000
	AE Secant	35:0' . . .	122078

[Table 15-2]

By this Proposition any secants you wish can be found.

Prop. 3. The Radius is to the sine of any arc you please: as the secant of the same to the tangent.

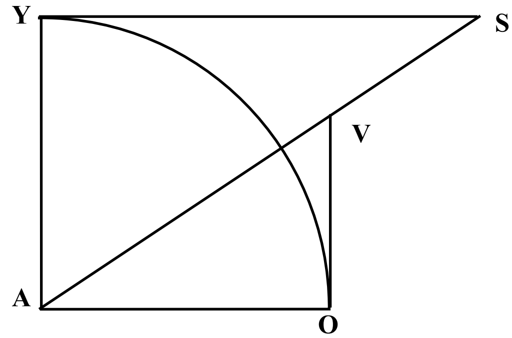
	AD Radius	. . . . ….	100000
Pro-	DB Sine	35:0'	57358
port.	AE Secant	35:0'	122078
	EC Tangent	35:0'	70021

[Table 15-3]

Prop.4. The radius is the mean proportional between tangents of these arcs as you please and of the complement.

	OV	Tangent	35:0' . . .	70021
Pro-	OA	Radius	. . . . ….	100000
port.	AY	Radius	. . . . ….	100000
	YS	Tangent	55:0' . . .	142812

[Table 15-3]



[Figure 15-2]

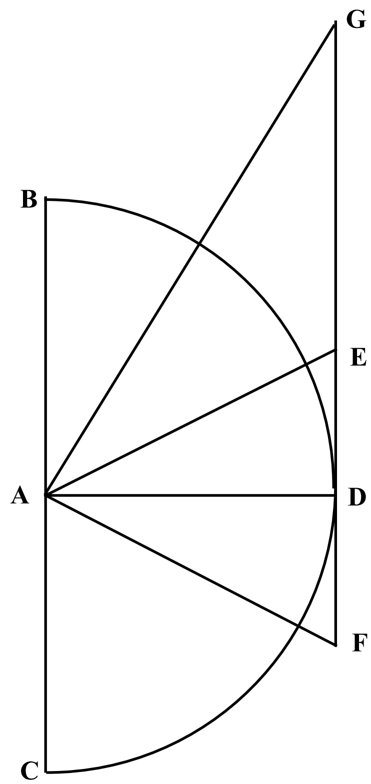
If by dividing the quadrant into 144 parts, by the first proposition of these, the tangents of half the quadrant or with the first 72 equal parts are found appropriately; the tangents of the remaining parts can be found, and the secants of all the others, by addition alone. As are demonstrated by the following Propositions.

Prop. 5 The secant of any arc you please, is equal to the [sum of the] tangents of the same arc and half of the complement.

Let the angle EAD be 23:0', and with the line GEF tangent to the periphery in the point D. GE, EF are taken equal to the line EA. GAF will be right, and DAF, EGA, EAG are equal among themselves, and EAG half the compliment [of EAD] EAB. But EF is equal (from the construction of the line AE to the secant of the angle EAD) to [the sum of] the tangents ED of the angle EAD given, and DF of the angle FAD of half the complement EAD 23:0', EAB 67:0' the complement , DAF 33:30'. [1]

ED	Tangent	EAD	23:0' .	4244748
DF	Tangent	DAF	33:30'	6618856
AE	Secant	EAD	23:0'	10863604

[Table 15-4]



[Figure 15-3]

The secant of any arc you wish, added to the Tangent of the same, is equal to the tangent of the arc *Prop. 6* composed from the given arc and half of the complement. [As AE + ED = DG: Figure 15-3].

For let the angle EAD be 23:0', the complement 67:0'; half of the complement 33:30'. The arc composed is 56:30'

AE	Secant	23:0' .	10863604
ED	Tangent	23:0'	4244748
DG	Tangent	56:30'	15108352

[Table 15-5]

The tangent of any arc you wish taken from the secant of the same, there is left the tangent of half the complement. *Prop. 7.*

ED	Tangent	23:0'	4244748
AE	Secant	23:0'	10863604
DF	Tangent	33:30'	15108352

[Table 15-6]

The Tangent doubled of any arc you wish, by addition to the tangent of half the complement, is equal to the tangent of the arc composed from the given arc and from half of the complement. *Prop. 8.* [For 2ED + DF = EF + ED = GE + ED = GD].

For by *Prop. 5* the secant is equal to the [sum of] tangents of the same arc and of half the complement. And therefore by *Prop. 6* if twice the tangent of the given arc is added to the tangent of half the complement, the sum is equal to the tangent of the composed arc. Therefore with the given tangents, with the individual parts for convenience from the first half quadrant ; the tangents can be found for the remaining parts for all the quadrants, by addition alone, following this Eighth Proposition.

The Arc 1: 15'. The Complement 88:45'

		218201			458293512
Tangent 1$\underline{250}$	*Tangent* 1:15': 0"	218201	88$\underline{750}$	*Tangent* 88:45'0"	458293512
Tangent 44$\underline{375}$	*Tangent* 44:22':30"	9784180	0$\underline{625}$	*Tangent* 0: 37:30	109088
Tangent 45$\underline{625}$	*Tangent* 45:37:30"	10220582	89$\underline{375}$	*Tangent* 89:22:30	916696112

The Arc 2: 30'. The Complement 87:30'

		436609			229037655
Tangent 2$\underline{500}$	*Tangent* 2:30':	436609	87$\underline{500}$	*Tangent* 87:30'	229037655
Tangent 43$\underline{750}$	*Tangent* 43:45'	9571917	1$\underline{250}$	*Tangent* 1: 15:	218201
Tangent 46$\underline{250}$	*Tangent* 46:15:	10446135	88$\underline{750}$	*Tangent* 88:45	458293511

[Table 15-7]

And by this method the tangents of the 144 separate parts of the quadrant are found; The secants of the other parts can be found *following Prop. 6* by addition alone. Indeed if the tangent of half the complement of that arc (of which the Secant is sought) has been given. As

The Arc given 1: 15. The Complement 88:45'

1250	*Tangent* 1:15: 0"	218210	87500	*Tangent* 88:45':0"	458293512
44375	*Tangent* 44:22:30	9784180	0625	*Tangent* 0: 37:30	109088
1250	*Secant* 1:15: 0	10002381	88750	*Secant* 88:45	458402600

[Table 15-8]

We have therefore a handy enough way for finding the tangents and secants from the 72 parts of the quadrant. It is possible to increase the number of these by quinquisection (as of the sines before): at first to 360, then to 1800, by the third to 9000, thus as with individual degrees the whole of the quadrant may be considered as composed from 100 tangents, and the same number of secants. The method of working is nearly the same as that which was expounded before on *page 38* [i.e. the interpolation scheme of Chapter 12.] From these, so many extra quantities are placed between; however in the first case with the correction of the differences for the sines as we used addition and subtraction equally, but with these it is fitting to use subtraction alone.

If we should wish to construct the canon to thousandths of degrees; the secants for the remaining parts of the quadrant, which were unable to be placed by the recent method, should be sought *by the second Proposition of this chapter*, for as with secants so with tangents for the 144 parts of the quadrant computed quickly. Then the number of these are increased as before to 720 parts, secondly to 3600 parts, and thirdly to 1800, and finally to 90000.

§15.1 Notes on Chapter 15.

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